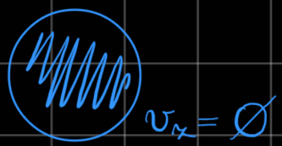
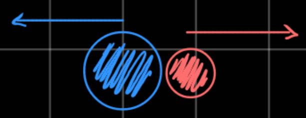
... od zadnjič

c) Kolikšen delež celotne E predstavlja odrivna E? $W_k$ produktov v procesu

Odrivna E

težiščni sistem



1) $E_x = m_x c^2$

2) $E_k = m_1 c^2 + \underbrace{T_1 + m_2 c^2 + T_2}_{\text{odrivna energija}}$

V našem primeru c) bomo računali $\frac{T_{Fe}}{E_\gamma} = ?$ - odrivna E (alt. način)

$$T_{Fe} = \frac{p^2}{2m_{Fe}} = \frac{(pc)^2}{2m_{Fe}c^2} \Rightarrow E_{odrivna} = \frac{E_\gamma^2}{2m_{Fe}c^2 E_\gamma} \doteq 10^{-4}$$

Čopič VI/15

$\beta^-$ razpad $\quad n \rightarrow p^+ + e^- + \bar{\nu}_e$

$m_n c^2 = 939{,}56\ \text{MeV}$

$m_p c^2 = 938{,}27\ \text{MeV}$

$m_e c^2 = 0{,}51\ \text{MeV}$

$m_{\bar{\nu}_e} = \emptyset$

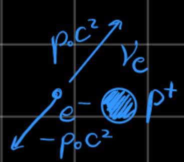
$v_e\ (max) = ?$

Primerjajmo 2 scenarija

a) proton miruje

začetek:


konec



b) el. antinevtrino miruje


a) Ohranitev $E$ in $p$

$$p_x^\mu c = (m_n c^2, 0)$$

$$\left.\begin{aligned} p_{e^-}^\mu c &= (E_{e^-}, -p_0 c) \\ p_{p^+}^\mu c &= (m_p c^2, \cancel{0}) \text{ miruje} \\ p_{\bar{\nu}_e}^\mu c &= (E_{\bar{\nu}_e}, p_0 c) \end{aligned}\right\} p_k^\mu c = (E_{e^-} + m_p c^2 + E_{\bar{\nu}_e}, 0)$$

Ohranitev četverca gibalne količine

$m_n c^2 = E_e + m_p c^2 + E_{\bar{\nu}_e}$, upoštevamo $E = \sqrt{(mc^2)^2 + (pc)^2}$

zanemarimo mirovno maso

$$m_n c^2 = \sqrt{(m_e c^2)^2 + (p_0 c)^2} + m_p c^2 + p_0 c \quad /^2$$

$$(m_n c^2)^2 + (m_p c^2)^2 + \cancel{(p_0 c)^2} - 2 m_n c^2 p_0 c - 2 m_n c^2 m_p c^2 + 2 m_p c^2 p_0 c = (m_e c^2)^2 + \cancel{(p_0 c)^2}$$

$$p_0 c = \frac{(m_e c^2)^2 - (m_n c^2)^2 - (m_p c^2)^2 + 2 m_n c^2 m_p c^2}{2(m_p c^2 - m_n c^2)} = 0'54\,\mathrm{GeV}$$

Op.: Vzeli smo ultrarelativistično limito in s tem zanemariti mirovno $E$. To ne bi bilo upravičeno, saj $m_e c^2 \ll E_{ekin} = p_0 c$

$$\left.\begin{aligned} pc &= m\gamma v c \\ E &= m\gamma c^2 \end{aligned}\right\} \quad \frac{pc}{E} = \frac{v}{c}, \quad E = \sqrt{(m_e c^2)^2 + (p_0 c)^2}$$

$$\Rightarrow \frac{v}{c} = 0{,}730 \Rightarrow v_e = 0{,}730c$$

b) Podobno, samo s kvadratno enačbo

# Dinamika jedrskih razpadov

$A \longrightarrow B$ razpad je naključni proces

Posledica teh trditev je razpadni zakon:
(ne vemo, kdaj jedro razpade, poznamo le verjetnost za to)

$$\frac{dN_A}{dt} = \frac{N_A}{\tau}, \quad \tau \text{ ... razpadni čas}$$

$$\int_{N_{A0}}^{N_A(t)} \frac{dN_A}{N_A} = \int_0^t \frac{dt}{\tau} \qquad \text{(izračun z Geiger-Nuttalovo enačbo)}$$

# razpadov $\propto$ # jeder (eksponentni proces)

$$\ln \frac{N_A(t)}{N_A(t=0)} = -\frac{t}{\tau} \Rightarrow N_A(t) = N_{A0}\, e^{-\frac{t}{\tau}}$$

Za razpolovni čas

$$\ln \frac{N_A(t)}{N_{A0}} = \log_2 \frac{N_A(t)}{N_{A0}} \ln 2 \Rightarrow N_A(t) = N_{A0}\, 2^{-\frac{t}{t_{1/2}}}$$

$t_{1/2} = \tau \ln 2$ ... razpolovni čas

Več razpadnih produktov

$$A \xrightarrow{\tau_1} B \qquad A \xrightarrow{\tau_2} C$$

Razpadni zakon $\quad \dfrac{dN_A}{dt} = -\dfrac{N_A(t)}{\tau_1} - \dfrac{N_A(t)}{\tau_2}$

Razpadna veriga

$$C \xrightarrow{\tau_1} A \xrightarrow{\tau_2} B$$

Razpadni zakon $\quad \dfrac{dN_A}{dt} = -\dfrac{N_A(t)}{\tau_2} + \dfrac{N_A(t)}{\tau_1}$

Opazi, da računamo za A, zato +, ko gre $C \to A$

$A_A$ ... aktivnost

$$A_A(t) = \left|\frac{dN_A}{dt}\right| = \frac{N_A(t)}{\tau} \quad \left[Bq = s^{-1}\right]$$

Zorko 5/13

$$X \longrightarrow {}^{124}Sb \xrightarrow{\tau} Y$$

$\tau_2 = 87$ dni

$A_X = 3{,}5 \cdot 10^9$ Bq in $\overbrace{A_X = \text{konst.}}^{\text{nenavadni pogoj}}$

$A_{Sb}(t=0) = 1{,}2 \cdot 10^9\,Bq$
$A_{Sb}(t=t_0) = 2{,}5 \cdot 10^9\,Bq$
$A_{Sb}(t=max) = max$

---

$t_0,\ t_{max} = ?$

Če določimo $A_{Sb}(t)$, lahko odgovorimo na obe vprašanji.

$$A_{Sb}(t) = \frac{N_{Sb}(t)}{\tau_2} \quad ; \quad \frac{dN_{Sb}}{dt} = -\frac{N_{Sb}}{\tau_2} + \underbrace{\frac{N_x}{\tau_1}}_{A_x}$$

$$\frac{dN_{Sb}}{dt} + \frac{N_{SB}}{\tau_2} = A_x$$ // rešimo DE $y' + Ay = f(x)$

- homogeni del

$$y' + Ay = \emptyset \Rightarrow y_H(x)$$

- partikularni del $y_p(x)$ (ali variacija konst.)

Rešitev: $y(x) = y_H + y_p$

Homogena rešitev $\frac{dN_{Sb}}{dt} + \frac{N_{Sb}}{\tau_2} = \emptyset$

$$\Rightarrow N^{h}_{Sb}(t) = K_1 e^{-\frac{t}{\tau_2}}$$

Partikularna rešitev $N^{p}_{Sb} = K_2 \neq N^{p}_{Sb}(t)$

nehomogenost je konst.

$$\Rightarrow N_{Sb}(t) = K_1 e^{-\frac{t}{\tau_2}} + K_2$$

$$\frac{dN_{Sb}}{dt} = K_1\left(-\frac{1}{\tau_2}\right)e^{-\frac{t}{\tau_2}}$$

Vstavimo v DE

$$-\frac{K_1}{\tau_2}\cancel{e^{-\frac{t}{\tau_2}}} + \frac{K_1}{\tau_2}\cancel{e^{-\frac{t}{\tau_2}}} + \frac{K_2}{\tau_2} = A_x$$

$$K_2 = A_x \tau_2$$

$K_1$ dobimo iz IC

$$A_{Sb}(t) = \frac{N_{Sb}(t)}{\tau_2} = \frac{K_1 e^{-\frac{t}{\tau_2}} + K_2}{\tau_2}$$

$$A_{Sb}(t=\emptyset) = \frac{K_1 + A_x\tau_2}{\tau_2} \Rightarrow K_1 = (A_{Sb}(t=\emptyset) - A_x)\tau_2$$

$$A_{Sb}(t) = \frac{1}{\tau_2}\left[(A_{Sb}(t=\emptyset) - A_x)\tau_2 e^{-\frac{t}{\tau_2}} + A_x\tau_2\right]$$

$$= (A_{Sb}(t=\emptyset) - A_x)e^{-\frac{t}{\tau_2}} + A_x$$

$$A_{Sb}(t_0) = (A_{Sb}(t=\emptyset) - A_x)e^{-\frac{t_0}{\tau_2}} + A_x$$

$$t_0 = (-\tau_2)\ln\left[\frac{A_{Sb}(t_0) - A_x}{A_{Sb}(t=\emptyset) - A_x}\right] = 72\text{ dni}$$

b) $A_{Sb}(t_{max}) = max$ ?

$$\frac{dA_{Sb}}{dt} = (A_{Sb}(t=\emptyset) - A_x)\left(-\frac{1}{\tau_2}\right)e^{-\frac{t}{\tau_2}}$$

Odvod nima ničel → maks. in minim. sta dosežena na robu definicijskega območja

$\Rightarrow$ Funkcija je naraščajoča $\rightarrow$ maks. bo dosežen, ko $t \rightarrow \infty$

in $A_{Sb}(t \rightarrow \infty) = A_x$

Zorko 5/14

$m_0(Ca) = 5\,mg$ $\qquad \tau = \dfrac{t_{\frac{1}{2}}}{\ln 2}$

$t_{\frac{1}{2}}(Ca) = 4{'}8$ dneva $\qquad \tau_1 = 6{'}92$ dneva

$t_{\frac{1}{2}}(Sc) = 3{'}43$ dneva $\qquad \tau_2 = 4{'}95$ dneva

$M(Ca) = 47\,g/mol$

$$^{47}Ca \xrightarrow[\beta]{t_{\frac{1}{2}}(Ca)} {}^{47}Sc \xrightarrow{t_{\frac{1}{2}}(Sc)} {}^{47}Ti$$

Začetna aktivnost pri $t=\emptyset$: samo Ca

$$A(t=\emptyset) = \frac{N_{Ca}(\emptyset)}{\tau_1} = \frac{m_0(Ca)\,N_A}{\tau_1\,M(Ca)} = 1{'}07 \cdot 10^{19}\,Bq$$

- $A_{t_{max}} = max$ ?

Potrebujemo $A_t$ in odvajamo: