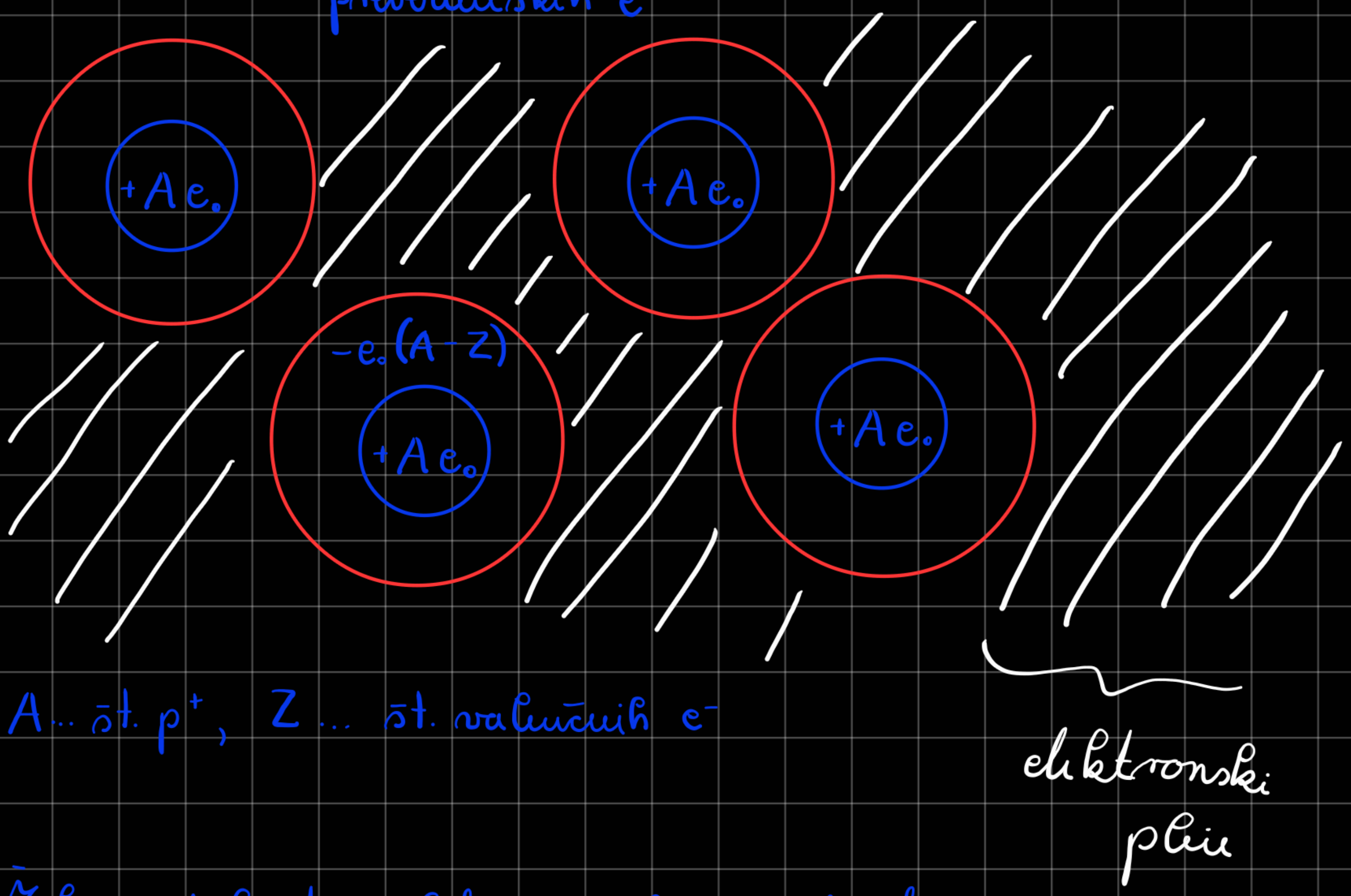
# Drudejev model

Model prevodnika, mostly wrong, nekatere enostavne lastnosti so super

prevodniških $e^-$




A... št. $p^+$, Z ... št. valenčnih $e^-$

Želimo določiti gibalno enačbo za el. plin.

$$d\vec{p} = \vec{F}dt$$

$e^-$ trkajo z jedri, verjetnost za trk $\frac{dt}{\tau}$, $\tau$... relaksacijski čas

Trke obravnavamo kot prožne

$t \neq 0$: $\langle\vec{p}(t)\rangle = 0$, či $\vec{E} = 0$, $e^-$ se gibljejo v vse smeri, nobena ni privilegirana, zato je povp. $\emptyset$

$$\langle\vec{p}(t)\rangle \neq 0 \quad \vec{E} \neq 0$$

$t = dt$

$$\langle p(t+dt)\rangle = \underbrace{\left(1-\frac{dt}{\tau}\right)}_{\text{verjetnost, da ni bilo trka}} \overbrace{\left(\underbrace{\langle\vec{p}(t)\rangle}_{\text{začetna } \langle p\rangle} - \underbrace{e_0\vec{E}\,dt}_{\text{sunek}}\right)}^{\text{za toliko se spremeni } \vec{p}} + \underbrace{\frac{dt}{\tau}\cdot\emptyset}_{\text{verjetnost, da trk je, } \langle\vec{p}\rangle \text{ se spremeni (?)}}$$

negativen predznak

$$O(dt^2) \doteq 0$$

$$\frac{\langle p(t+dt)\rangle - \langle p(t)\rangle}{dt} = -e_0E - \frac{\langle\vec{p}(t)\rangle}{\tau}$$

$$\Rightarrow \boxed{\frac{d\langle p(t)\rangle}{dt} = -e_0E - \frac{\langle\vec{p}(t)\rangle}{\tau}}$$

Po nekaj časa: stacionarno stanje

$$\frac{d\langle\vec{p}(t)\rangle}{dt} = 0$$

$\Rightarrow \quad \langle\vec{p}(t)\rangle = -e_0E$ // usmerjeno gibanje $e^-$

Zanima nas el. tok $I \Rightarrow$ gost. el. toka $j$

$$j = n(-e_0)\langle\vec{v}\rangle, \quad n = \frac{N}{V}$$

klasično $\langle\vec{p}\rangle = m\langle\vec{v}\rangle$

$$j = \frac{ne_0^2\tau}{m}\vec{E}$$ // pove nam, kako se mora odzove na $\vec{E}$

El. prevodnost $\sigma$ : $\boxed{\vec{j} = \sigma \vec{E}}$

$\boxed{\sigma_0 = \frac{n e_0^2 \tau}{m}}$ statična (specifična) el. prevodnost

5/24 (Čopič skripta)

$E = 1 V/m$

$\mu = 0'0032 \, m^2/Vs$ // gibljivost $\mu$ : odziv mosi na $\vec{E}$

$\boxed{\langle \vec{v} \rangle = \mu \vec{E}}$

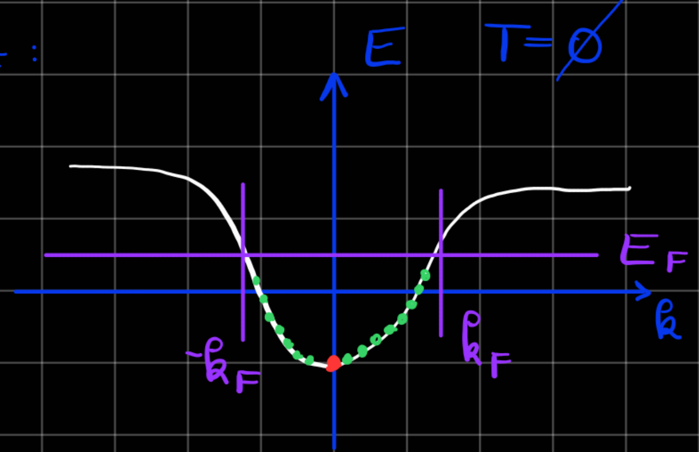
$E_F = 7'0 \, eV$ // Fermijeva en.

$\langle \vec{v} \rangle = ?$

$\frac{\langle \vec{v} \rangle}{\vec{v}_F} = ?$

$\langle \vec{v} \rangle = 0'0032 \, m/s$

$E_F$ :




$k_l = \frac{2\pi}{Na} l \, , \; l \in \mathbb{Z}$

- $e^-$ zasedajo mesta

1. $e^-$ gre v rdečo točko, polnjenje stanj

Najvišja en, ki jo $e^-$ zasedajo pri $T=\emptyset$ je $E_F$

Pri $T\neq\emptyset$ imamo še termično ekstrakcijo (ker $F=E-TS$)

$$\vec{p}=\hbar\vec{k}=m\vec{v} \Rightarrow k_F=\frac{m v_F}{\hbar}$$

Splošna en. $E=\frac{\hbar^2 k^2}{2m_e} \Longrightarrow$ Fermi, dodamo indikse

$$E_F=\frac{\hbar^2 k_F^2}{2m_e}, \quad m_e=9{,}1\cdot 10^{-31}\,kg$$

$$k_F=\sqrt{\frac{2m E_F}{\hbar^2}} \Rightarrow v_F=\frac{\hbar k_F}{m}$$

$$\frac{\langle\vec{v}\rangle}{v_F}=2{,}0\cdot 10^{-9}, \quad v_F \gg \langle\vec{v}\rangle$$

Za to, da dosežemo $v_F$, mora biti $k_B T$, $T=10\,000K$

Nerealističen model

Pri $T=0$ so s stanja z najmanjšo en. zasedena, pri $T\neq 0$, so stanja razpacana zaradi entropije (zelene pikice)

5/23

$\rho_{Na}=0{,}97\,g/cm^3$

$\rho_{Cu}=8{,}9\,g/cm^3$

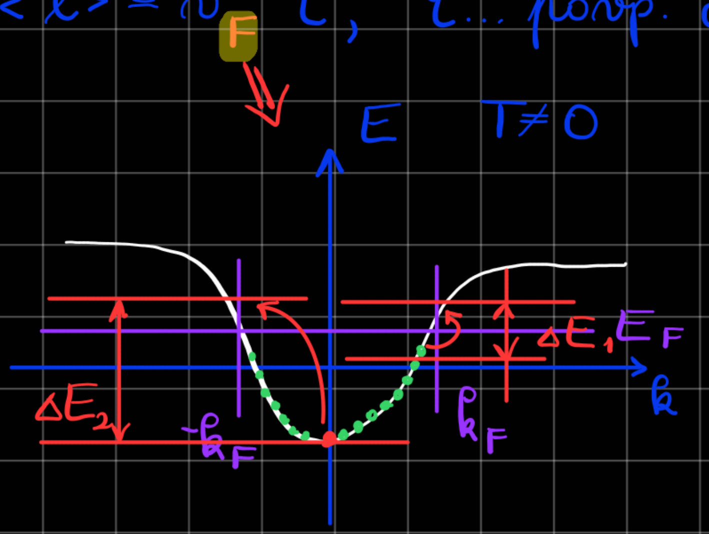
$Z_{Cu} = Z_{Na} = 1$

$\sigma_{Cu} = 5{,}9 \cdot 10^5 / \Omega m$

$\sigma_{Na} = 2{,}2 \cdot 10^5 / \Omega m$

Povp. prosta pot

$\langle l \rangle = v_F \tau$, $\tau$ ... povp. čas do trka




$\Delta E_2 \ll \Delta E_1$

$e^-$ v bližini $E_F$ se sipajo $(\Delta E_1)$, saj so jim prosta mesta iz prevodnega pasu blizu

$\sigma = \frac{n e_0^2 \tau}{m}$, $\rho \to n$ – prevodniški el. !

$\rho = \frac{m}{V}$, $m_e \ll m_p$, $m_j$... masa jedra

$\rho = \frac{m_j}{V} = \frac{n M}{V}$, $n$... množina snovi, $M$... molska masa

$= \frac{N}{N_A} \frac{M}{V}$, $\frac{N}{V} = n_{at}$

$n_{at} = \frac{\rho N_A}{M} \Rightarrow n_{el} = Z n_{at}$

$n_{el,Na} = 2'5 \cdot 10^{28} / m^3$

$n_{el,Cu} = 4'54 \cdot 10^{28} / m^3$

$N_{st} = \sum_{\vec{k}} 2$ // stanja

$\downarrow$ (degeneracija)

$\forall \vec{k}$ ima dva dovoljena stanja $e^-$

$N_{st,zas} = \sum_{|\vec{k}| \leq k_F} 2 = N_e$ // zasedena stanja (meja je $k_F$)

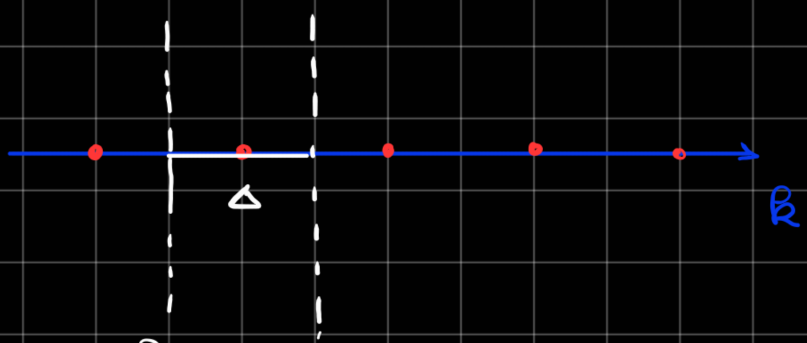
$\downarrow$

$T = 0$

Želimo $\sum \to \int \frac{d^3k}{\Delta}$, $\Delta$ - Volumen v k-prostoru, ki pripada določenemu $\vec{k}$

Določitev $\Delta$ v 1D

$k_l = \frac{2\pi}{L} l$



$\Delta = \frac{2\pi}{L}$

v 3D: $k_{l_x, l_y, l_z} = 2\pi \left( \frac{l_x}{L_x}, \frac{l_y}{L_y}, \frac{l_z}{L_z} \right)$

$$\Delta = \frac{(2\pi)^3}{V}, \quad V = L_x \cdot L_x \cdot L_y$$

$$N_{el} = 2\int_0^{\vec{k}} \frac{d^3k}{\frac{2\pi}{V}}, \quad \vec{k} \leq |k_F|$$

v prostoru se tvori krogla

kartezične koordinate → sferične koordinate

$$N_{el} = \frac{2V}{(2\pi)^3} \frac{4}{3} \pi k_F^3$$

$$n_{el} = \frac{N_{el}}{V} = \frac{k_F^3}{3\pi^2} \Rightarrow k_F = \sqrt[3]{3\pi^2 n_{el}}$$

↳ vsi $e^-$ ! napravu enačbi za $\tau$

$$n = \frac{A \rho N_A}{M}, \quad A \ldots \text{št. } p^+$$

$$v_F = \frac{\hbar k_F}{m}$$

$$v_{F,Cu} = 1{,}6 \cdot 10^6 \, m/s \Rightarrow \langle l \rangle_{Cu} = 40 \, nm$$

$$v_{F,Na} = 1{,}0 \cdot 10^6 \, m/s \Rightarrow \langle l \rangle_{Na} = 30 \, nm$$

4.11 Skripta Zorko

$\omega = 30 \, MHz$

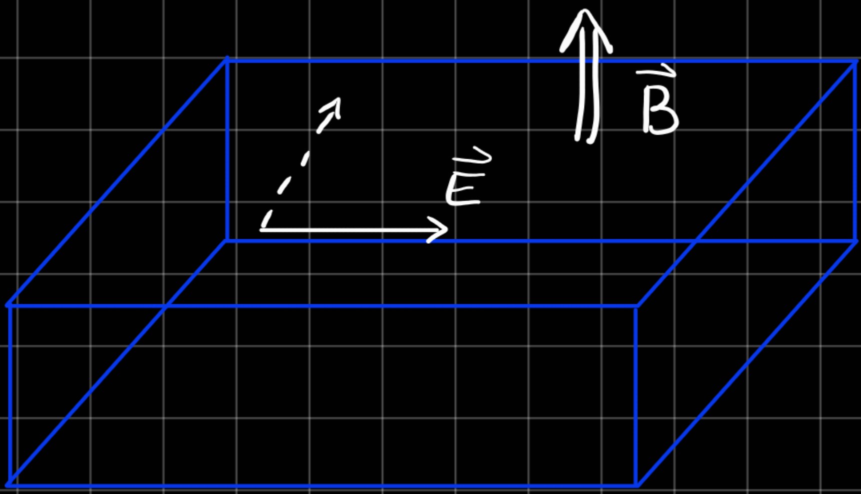
$M_{Pt} = 195{,}1 \, kg/kmol$

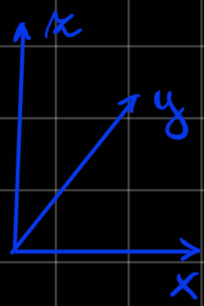
$B_0 = 2 \, mT$

$\tau = 5{,}1 \cdot 10^{-15} \, s$

$\rho_{Pt} = 21{,}5 \, kg/m^3$

$Z_{Pt} = 1$




$$B = B_0 e_z = \begin{bmatrix} 0 \\ 0 \\ B_0 \end{bmatrix}$$

$$\vec{E} = \begin{bmatrix} E_0 e^{-i\omega t} \\ E_0 e^{-i(\omega t + \delta)} \\ 0 \end{bmatrix}, \quad \delta = -\frac{\pi}{2}$$

$$\vec{E} = E_0 e^{-i\omega t} \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix}$$

$$\vec{j} = \sigma \vec{E} = \vec{j}_0 e^{-i\omega t}$$

Iščemo prevodnost:

$$\frac{d\langle \vec{p}(t)\rangle}{dt} = -\frac{\langle \vec{p}(t)\rangle}{\tau} - e_0\left(\vec{E} + \langle \vec{v}\rangle \times \vec{B}\right)$$

$\frac{dj}{dt}$ nas zanima

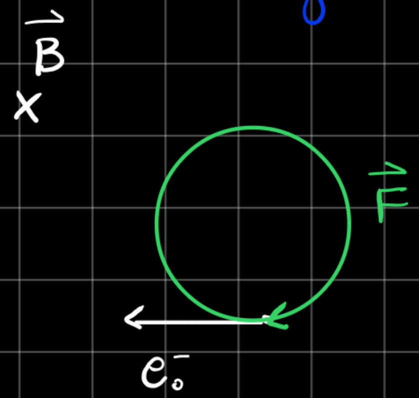
$$\vec{j} = -e_0 n \langle \vec{v} \rangle \Rightarrow \langle \vec{v} \rangle = -\frac{j}{n e_0}$$

$$\langle p \rangle = m \langle \vec{v} \rangle = -\frac{m \vec{j}}{n e_0}$$

$$\Rightarrow \tau \frac{d\vec{j}}{dt} = -\vec{j} + \overbrace{\frac{n e_0^2 \tau}{m}}^{\sigma} \vec{E} - \overbrace{\frac{e_0 \tau}{m} \vec{j} \times \vec{B}}^{\omega_c \tau \vec{j} \times \vec{e}_z}$$

uvedimo: • statička provodnost $\sigma = \frac{n e_0^2 \tau}{m}$

• ciklotronska frekvencija $\omega_c = \frac{v}{r}$




$$F_m = F_{cp}$$

$$e_0 \cancel{v} B_0 = \frac{v^{\cancel{2}}}{r} m$$

$$\boxed{\omega_c = \frac{e_0 B_0}{m}}$$

$$\vec{j} \times e_z = \begin{bmatrix} j_{0y} \\ -j_{0x} \\ \cancel{0} \end{bmatrix}$$

$$\vec{j} = j_0 e^{-i\omega t} \Rightarrow \frac{d\vec{j}}{dt} = \vec{j}_0 (-i\omega) e^{-i\omega t}$$

Po komponentah zapis:

x-smer: $-i\omega\tau j_{0x} = -j_{0x} + \sigma_0 E_0 - \tau\omega_c j_{0y}$ (1)

y-smer: $-i\omega\tau j_{0y} = -j_{0y} + i\sigma_0 E_0 + \tau\omega_c j_{0x}$ (2)

z smer: $j_z = \emptyset$

(1) + i (2): $\underbrace{(1 - i(\omega + \omega_c)\tau)}_{\text{ne more biti nič}} \underbrace{(j_{0x} + i j_{0y})}_{=0} = \emptyset$ $\Rightarrow j_{0y} = i j_{0x}$

// vstavimo v 1. enačbo

(1) $[1 - i(\omega - \omega_c)\tau] j_{0x} = \sigma_0 E_0$

spomnimo se $\vec{j} = \sigma_0 \vec{E}$

$\vec{E} \propto e^{-i\omega t}$

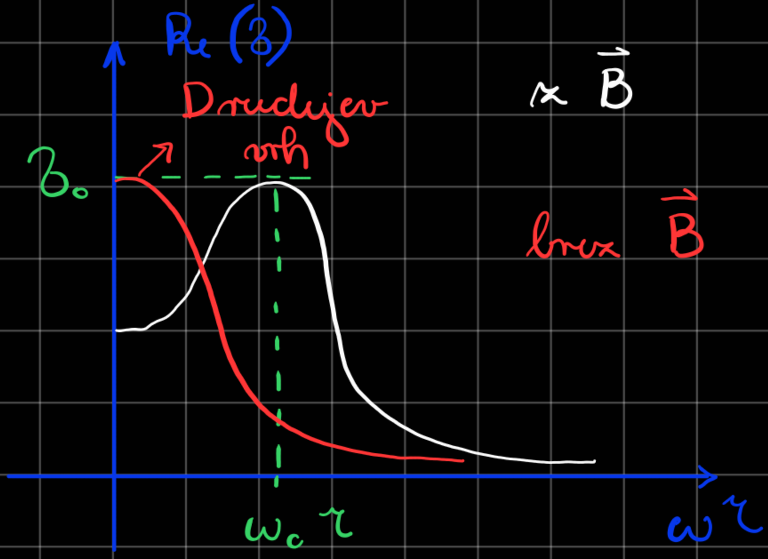
$\vec{j} \propto e^{-i\omega t}$

$$j_{0x} = \sigma_0 E_{0x} \Rightarrow \sigma = \frac{\sigma_0}{1 - i(\omega - \omega_c)\tau} = \underbrace{\mathrm{Re}(\sigma)}_{\text{odziv okoliških } e^-} + i\underbrace{\mathrm{Im}(\sigma)}_{\text{pove nam koliko en. se izgubi zaradi absorpcije}}$$

= optična prevodnost

// imenovalec realiziramo dispacija en.

$$\sigma = \frac{1 + i(\omega + \omega_0)\tau}{1 + (\omega + \omega_0)^2\tau^2}\sigma_0$$




$\propto \vec{B}$

brez $\vec{B}$

Popravek