Nadaljevanje od zadnjič

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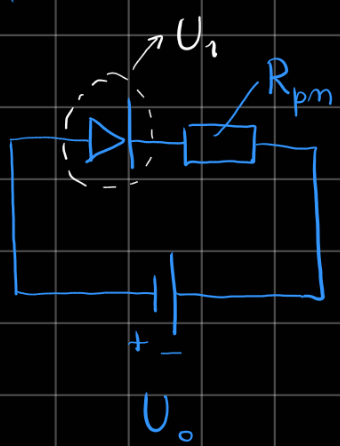
1) Najprej vzamemo idealno diodo

$$I = I_0 \left(e^{-\beta e_0 U} - 1\right)$$

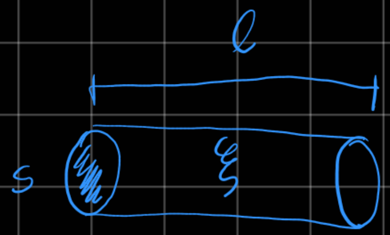
// obrnemo za U_1

$$U_1 = \frac{1}{\beta e_0} \ln\left[\frac{I}{I_0} + 1\right] = 0{,}24\,V$$

2)

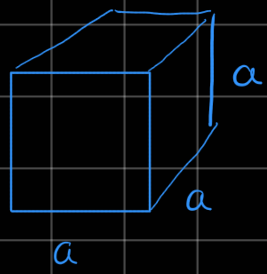


$U_0 = U_1 + IR$ // zanima nas R, ker U_0 in U_1 poznamo



$$R = \frac{\zeta l}{S}$$

// za kocko je



$$R = \frac{\zeta a}{a^2} = \frac{1}{\sigma a}$$

$$R_{pn} = R_p + R_n = \frac{1}{a}\left[\frac{1}{R_p} + \frac{1}{R_n}\right] = 7\,\Omega$$

U_0 je napetost vira, ki ga moramo priklopiti

$$U_0 = \overset{0'24V}{U_1} + R\underset{10mA}{I} = 0'31V$$

Jedra

Gostota v jedru je večinoma konst. Vezavna en. je premo sorazmerna s številom gradnikov ($2\times$ št. gradnikov $\rightarrow 2\times E_v$)

Velikost je reda ~fm

jedro: (p^+ n^0)

A ... oznaka št. gradnikov

Jedro elementa $^{A}_{Z}X$

A → masno št. ($A = Z + N$, N je # nevtronov)

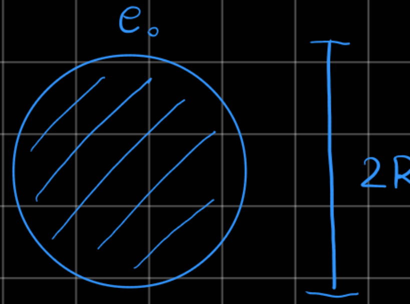
Z → vrstno št. (# protonov)

// predpostavimo, da je jedro iz trdne snovi (ni gradniki so tudi
// empirično lahko z $\rho = $ konst. zapišemo radij jedra kroglic)

$$r_j = r_1 A^{\frac{1}{3}} \quad , \quad r_1 = 1'1 \text{ fm}$$

Elektrostatika

Čopič VI/2



// naboj je enakomerno razmazan

$E_{el} = ?$ $\quad \varphi(r=0) = ?$

// z Gaussovim zakonom določimo $\vec{E}$ zunaj in znotraj krogle

1) Gauss $\rightarrow \vec{E}(\vec{r})$

2) $\vec{E}(\vec{r}) \rightarrow \varphi(\vec{r})$ // potencial

3) $\varphi(\vec{r}) \rightarrow E_{el}$ // elektrostatska en.

1) $e = \varepsilon \oint \vec{E}\, d\vec{S}$ $\quad d\vec{S}$ so sfere (smo v 3D)

$$E(r) = \frac{e(r)}{4\pi r^2 \varepsilon_0}$$

a) $r \leq R$

$e = \rho_e V$ // č je $e = e(r)$ integral $\int_V \rho(r)\, d^3\vec{r}$

$$e = \underbrace{\frac{e_0}{\frac{4}{3}\pi R_0^3}}^{\rho_e} \underbrace{\frac{4}{3}\pi r^3}_{V}$$

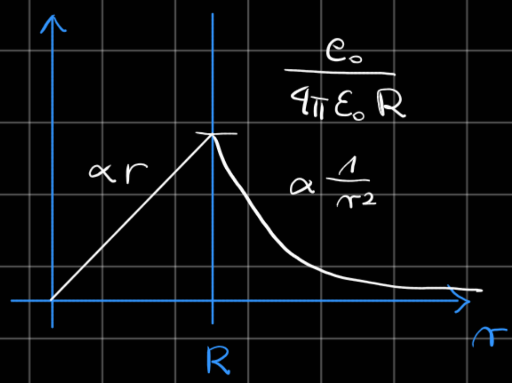
$$E_{<R}(e) = \frac{e_0}{4\pi \varepsilon_0 R^3} r$$

b) $r > R$

točkast naboj, $e =$ konst.

$$E(r) = \frac{e(r)}{4\pi \varepsilon_0 r^2} \Rightarrow E_{>R}(e) = \frac{e_0}{4\pi \varepsilon_0 r^2}$$

$$E(r) = \begin{cases} E_{<R}, & r \leq R \\ E_{>R}, & r > R \end{cases}$$



2) potencial $\varphi(r) = ?$

$$\vec{E}(\vec{r}) = -\nabla \varphi(\vec{r})$$

$$\varphi(\vec{r}) = -\int E(\vec{r})\, d\vec{s}$$

// ∇ v kartezičnih $\longrightarrow$ // ∇ v sfernih

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \longrightarrow \nabla = \frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial \vartheta}\hat{e}_\vartheta + \frac{1}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\hat{e}_\varphi$$

// dela $\hat{e}_\vartheta$ in $\hat{e}_\varphi$ odpadeta, ker $\vec{E} = E(\hat{e}_r)$

a) $r > R$

// nedoločeni int

$$\varphi_{>R}(r) = -\int \frac{e_0}{4\pi\varepsilon_0 r^2}\,dr = \frac{e_0}{4\pi\varepsilon_0 r} + A$$

b) $r \leq R$

$$\varphi_{<R}(r) = -\int \frac{e_0 r}{4\pi\varepsilon_0 R^3}\,dr = -\frac{e_0 r^2}{8\pi\varepsilon_0 R^3} + B$$

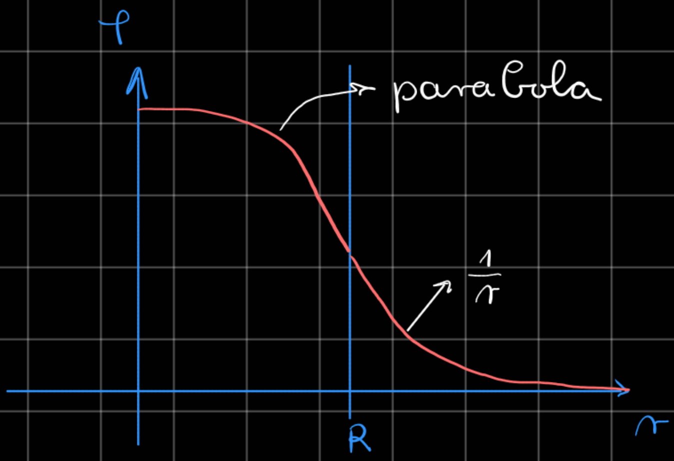
a.1) $\varphi(r \to \infty) \longrightarrow 0 \Rightarrow A = \emptyset$

b.1) $\varphi_{<R}(r = R) = \varphi_{>R}(r = R)$

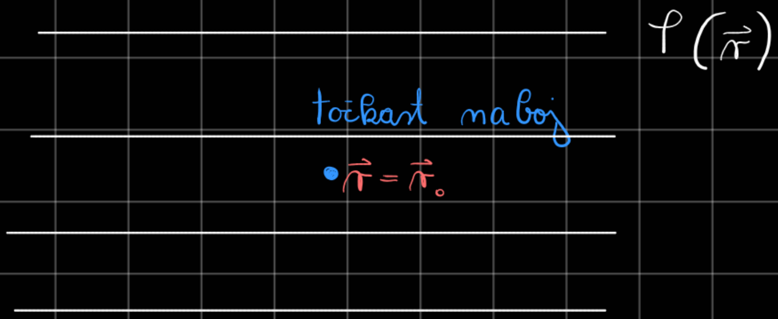
$$-\frac{e_0 r^2}{8\pi\varepsilon_0 R} + B = \frac{e_0}{4\pi\varepsilon_0 R}$$

$$\Rightarrow B = \frac{3 e_0}{8\pi\varepsilon_0 R}$$

$$\varphi(r = 0) = \frac{3 e_0}{8\pi\varepsilon_0 R} \qquad \varphi_{>R}(r) = \frac{e_0}{8\pi\varepsilon_0 R}\left[3 - \left(\frac{r}{R}\right)^2\right]$$



3) Elektrostatska energija



Če nimamo točkastega naboja, ampak netočkastega, razbijemo na infinitezimalne dele

$$E_{el} = e\,\varphi(\vec r = \vec r_0)$$

// za jedro $\quad E_{el} = \frac{1}{2}\int_V \rho(\vec r)\, de$

parska interakcija, deluje in izračunamo je na par, nas pa zanima samo za kroglico

$$e(r) = \frac{e_0 r^3}{R^3} \Rightarrow \frac{de}{dr} = \frac{3e_0 r^2}{R^3}$$

$$E_{el} = \frac{1}{2}\,\frac{e_0}{8\pi\varepsilon_0 R}\,\frac{3e_0}{R^3}\underbrace{\int_0^R dr\, r^2\left[3-\left(\frac{r}{R}\right)^2\right]}_{\frac{4}{5}}$$

$$x = \frac{r}{R} \qquad \frac{dx}{dr} = \frac{1}{R}$$

$$= \frac{3e_0^2}{16\pi\varepsilon_0 R^4}\left[3\,\frac{x^3}{3}\Big|_0^1 - \frac{x^5}{5}\Big|_0^1\right]$$

$$E_{el} = \frac{3 e_0^2}{20\pi \varepsilon_0 R}$$

// za občutek $R \sim fm \Rightarrow E_{el} \sim MeV$

$$E_{el} = \frac{3}{5} \underbrace{\boxed{\frac{e_0^2}{4\pi \varepsilon_0 \hbar c}}}_{=\alpha} \frac{\hbar c}{R}$$

$\hbar c \rightarrow 197\ MeV\ fm$

$R \rightarrow \sim fm$

$\alpha \doteq \frac{1}{137}$ // konstanta fine strukture

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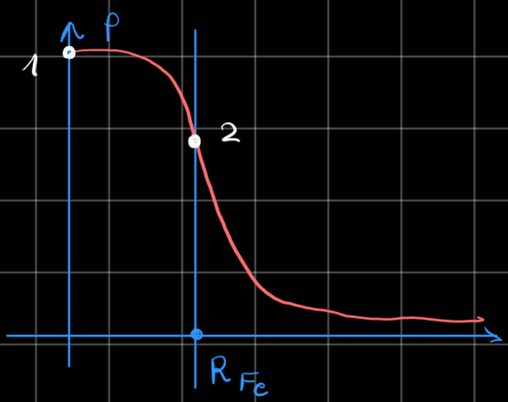
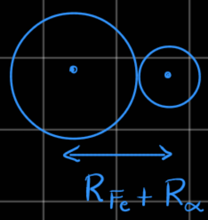
$\alpha \longrightarrow {}^{56}_{26}Fe$ // Iščemo kinetično en. α delca

$\alpha = {}^{4}_{2}He^{2+}$ // brez e^-

Skica

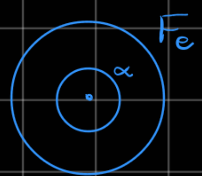


// pri 2 se jedri dotikata



// tukaj bomo vzeli potencial, ki je zunaj jedra

// pri 1 se središči prekrivata



$\vec{r}^{\,*}_{Fe} = \vec{r}^{\,*}_{\alpha}$

Fe:

$A = 56$, $Z = 26$

α:

$A = 4$, $Z = 2$

// empirična formula od prej $r_j = r_1 A^{\frac{1}{3}}$

$R_{Fe} = 4'21$ fm

$R_\alpha = 1'75$ fm

1) $E_1 = \rho(r=0)\cdot e_\alpha$ // predpostavimo, da je α točkasto

$$= \frac{3 Z_{Fe} Z_\alpha e_0^2}{8\pi \varepsilon_0 R_{Fe}} = \frac{3}{2} Z_{Fe} Z_\alpha \alpha \cdot \frac{\hbar c}{R_{Fe}} = 26'6 \text{ MeV}$$

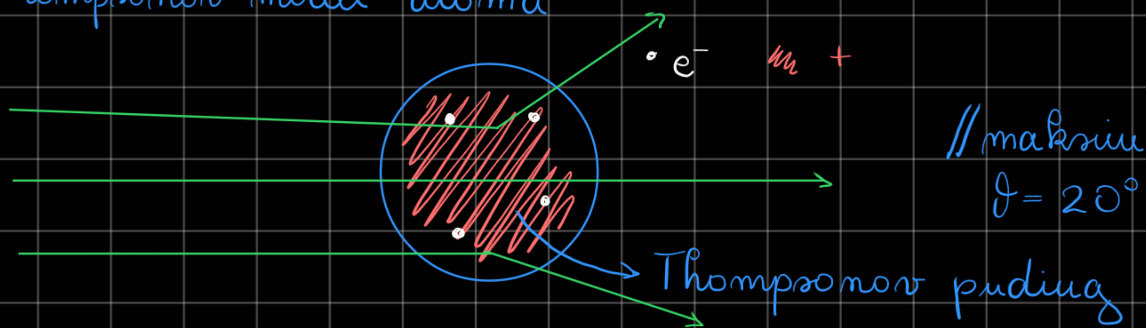
2) $E_2 = \dfrac{Z_{Fe} Z_\alpha e_0^2}{4\pi \varepsilon_0 (R_{Fe}+R_\alpha)} = 12'5$ MeV

Rutherfordovo sipanje

Pred Rutherfordom je bil Thompsonov puding (homogena mas
s točkastimi naboji)

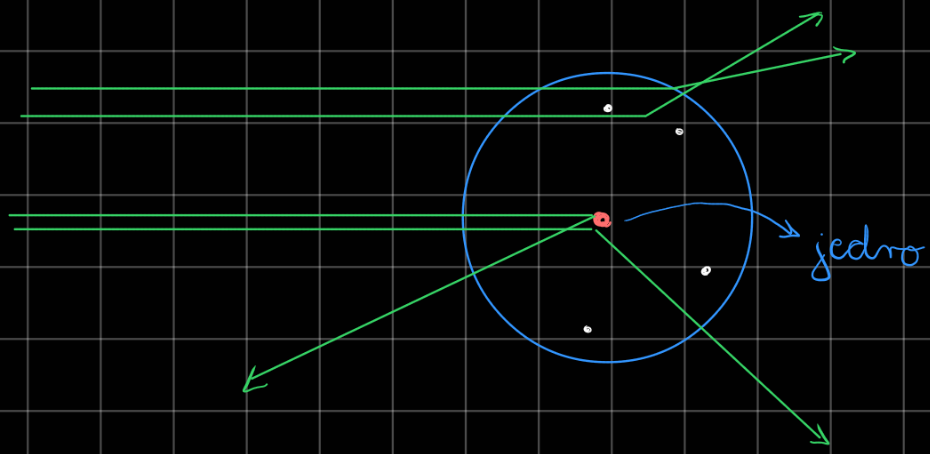
Po poskusu: jedro je majhen del atoma

Thompsonov model atoma



// maksimalni kot sipanja
$\vartheta = 20°$

Delci so se v eksp. sipali tudi za π. Rutherfordov model:



// zelene puščice so poti sipanja

Centralni potencial (sila kaže v center) $\Rightarrow$ ohranitev $\vec{L}$