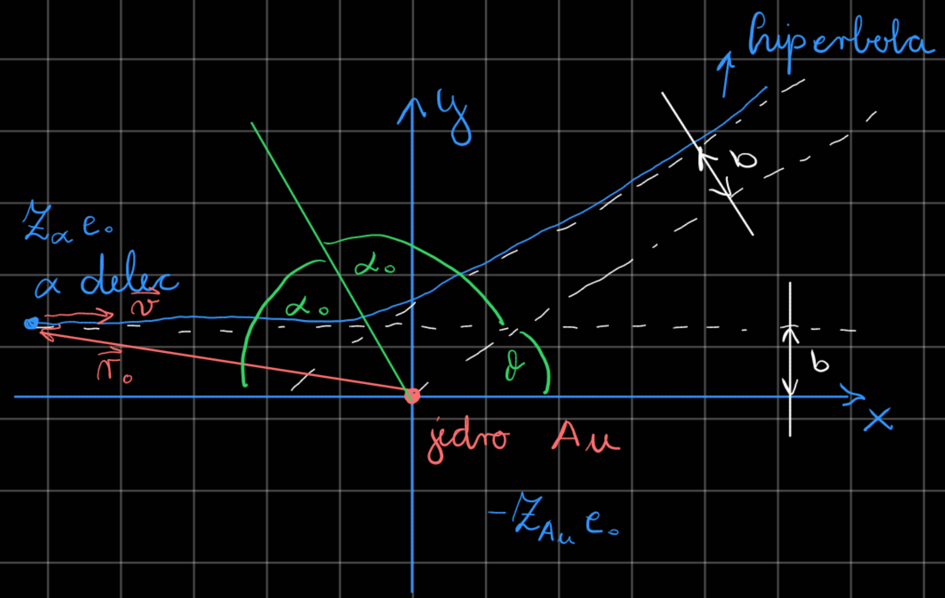
Rutherfordovo sipanje



Težiščni sistem z reducirano maso in velja

$$m_\alpha << m_{Au}$$

$$\Rightarrow \mu \doteq m_{Au} \quad \text{// reducirana masa}$$

$$\vec{v} = (v_0, 0)$$

Zanima nas verjetnost za sipanje (izpeljali bomo sipalni presek)

1) Gibanje v centralnem potencialu

$$\vec{F} \parallel \vec{r} \Rightarrow \vec{M} = \emptyset \quad \text{// navor}$$

$$\vec{M} = \emptyset \Rightarrow \vec{\Gamma} = \text{konst.} \qquad \frac{d\vec{\Gamma}}{dt} = \vec{M} \quad \text{// konst. gibanja}$$

$$\vec{\Gamma} = \vec{r} \times \vec{p}$$

$$\vec{p} = m\vec{v} = m\dot{\vec{r}}$$

Definiramo $\vec{r} = (-r\cos\varphi, r\sin\varphi, 0)$

Velja $r = r(t)$ in $\varphi = \varphi(t)$ in tako je

$$\dot{\vec{r}} = (-\dot{r}\cos\varphi + r\sin\varphi\,\dot{\varphi},\ \dot{r}\sin\varphi + r\cos\varphi\,\dot{\varphi},\ 0)$$

$$\Rightarrow \vec{\Gamma} = m \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} = m\left(0, 0, r_x v_y - r_y v_x\right)$$

$$\Gamma_z = m\left[-r\dot{r}\cancel{\sin\varphi\cos\varphi} - r^2\cos^2\varphi\,\dot{\varphi} + r\dot{r}\cancel{\sin\varphi\cos\varphi} - r^2\sin^2\varphi\,\dot{\varphi}\right]$$

$$= -m r^2 \dot{\varphi}$$

Želimo povezati ~~z~~ b in v_0 (Γ):

$$\Gamma(t=\emptyset) = \vec{r}(t=\emptyset) \times \vec{p}(t=\emptyset) = m v_0 b \quad // \vec{r}_\parallel \times \vec{p} = 0$$

$$\vec{r} = \vec{r}_\perp + \vec{r}_\parallel \qquad |\vec{r}_\perp| = b$$

Tako velja $\cancel{m} v_0 b = -\cancel{m} r^2 \dot{\varphi} \Rightarrow \dot{\varphi} = -\frac{v_0 b}{r^2}$

2) Newtonov zakon v y-smeri

$$m\frac{dv_y}{dt} = F_y \qquad // \text{ poznamo } \dot{\varphi}\text{, zato vrižno odvajamo}$$

$$m\frac{dv_y}{d\varphi}\frac{d\varphi}{dt} = \frac{Z_\alpha Z_{Au} e_0^2}{4\pi\varepsilon_0 r^2}\cdot\sin\varphi$$

$$\frac{d\varphi}{dt} = \dot{\varphi} = \frac{v_0 b}{r^2}$$

$$\frac{dv_y}{d\varphi} = \frac{Z_\alpha Z_{Au} e_0^2}{4\pi\varepsilon_0 v_0 b m}\sin\varphi$$

// zanima nas končno spremenjanje hitrosti $\Rightarrow$ integracija

$$\int_0^{v_y} dv_y = \frac{Z_\alpha Z_{Au} e_0^2}{4\pi \varepsilon_0 v_0 b m} \int_0^{\varphi} \sin\varphi \, d\varphi$$

$$v_y(\varphi) = \frac{Z_\alpha Z_{Au} e_0^2}{4\pi \varepsilon_0 v_0 b m} \left[1 - \cos\varphi \right]$$

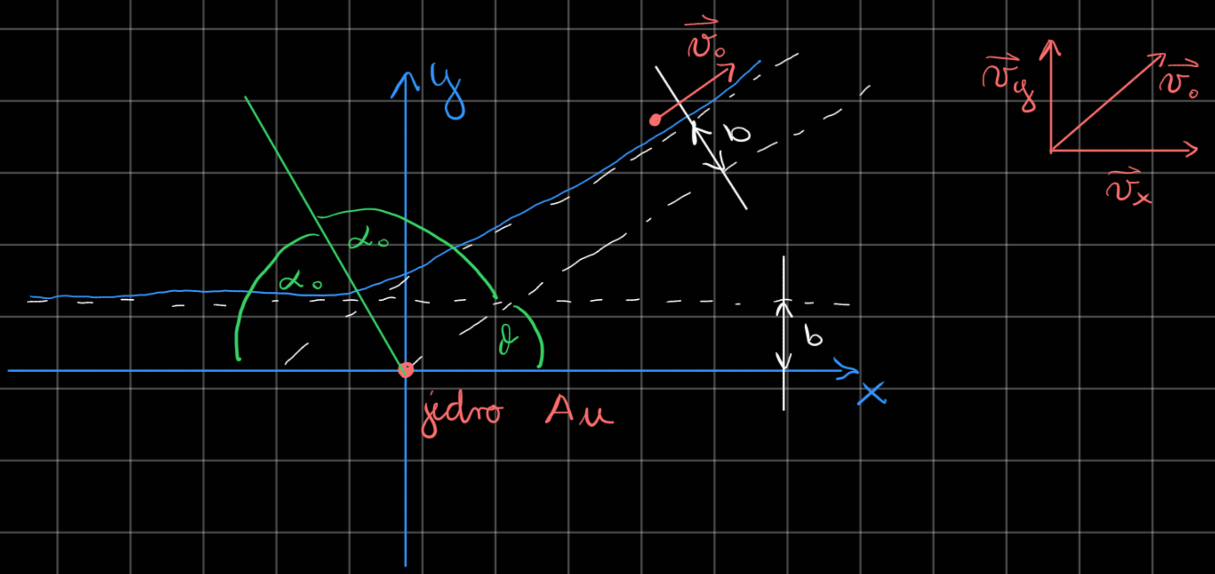
// preidemo iz φ na ϑ, ker nas zanima kot sipanja

Po sipanju : $\varphi \rightarrow \varphi = \pi - \vartheta$

$$\Rightarrow \cos(\pi - \vartheta) = -\cos(\vartheta)$$

$$v_y(\vartheta) = \frac{Z_\alpha Z_{Au} e_0^2}{4\pi \varepsilon_0 v_0 b m} \left[1 + \cos\vartheta \right]$$

// nahajamo se dimo



$$v_y = v_0 \sin\vartheta$$

Zanima nas parameter b : $b = \underbrace{\boxed{\frac{Z_\alpha Z_{Au} e_0^2}{4\pi \varepsilon_0 m v_0^2}}}_{K} \; \frac{1+\cos\vartheta}{\sin\vartheta}$

// dvojni koti: $\cos\vartheta = \cos\left(2\frac{\vartheta}{2}\right) = \cos^2\left(\frac{\vartheta}{2}\right) - \sin^2\left(\frac{\vartheta}{2}\right)$

$$\sin\vartheta = 2\sin\left(\frac{\vartheta}{2}\right)\cos\left(\frac{\vartheta}{2}\right)$$

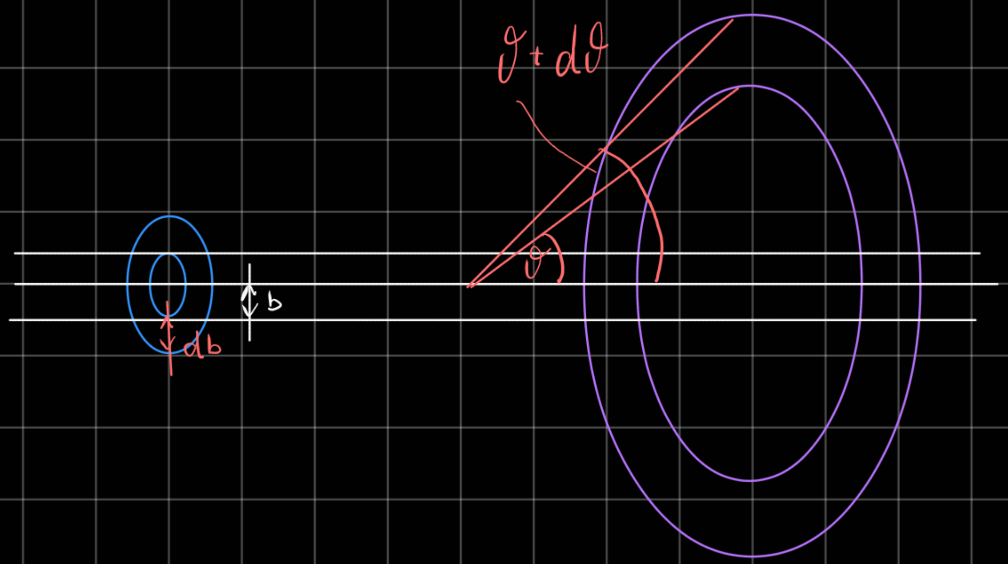
$$1 = \cos^2\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2}$$

$$b = K\,\frac{\cos\left(\frac{\vartheta}{2}\right)}{\sin\left(\frac{\vartheta}{2}\right)} = \frac{K}{\operatorname{tg}\left(\frac{\vartheta}{2}\right)}$$

$$K = \frac{Z_\alpha Z_{Au}\, e_0^2}{4\pi\varepsilon_0 m v_0^2} = Z_\alpha Z_{Au} \overbrace{\boxed{\frac{e_0^2}{4\pi\varepsilon_0 \hbar c}}}^{\alpha} \hbar c\, \frac{1}{\frac{m v_0^2}{2}\cdot 2} = \frac{Z_\alpha Z_{Au}\, \alpha\, \hbar c}{2E_{kin}}$$

3) Porazdelitev po kotih

$$\frac{d\overset{\text{sipalni presek}}{\sigma}}{d\Omega}\, d\Omega = \frac{\#\text{ sipanih delcev v intervalu } [\Omega, \Omega + d\Omega] \text{ na časovno enoto}}{\text{vhodno intenziteto/tok}}$$



Ohranitev # delcev

$$I = \frac{dN}{dS} \Rightarrow I\,dS = I\,2\pi b\,db \quad // \text{ # vpadnih delcev}$$

$$I \frac{d\sigma}{d\Omega} d\Omega \quad // \text{ # po sipanju}$$

$$\hookrightarrow d(\cos\vartheta)\,d\varphi = 2\pi \sin\vartheta\,d\vartheta$$

ENAKA

Upoštevamo enakost

$$\cancel{I}\,db\,\cancel{2\pi}\,b = \cancel{I}\frac{d\sigma}{d\Omega}\,\cancel{2\pi}\sin\vartheta\,d\vartheta$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\vartheta}\frac{db}{d\vartheta} = K\frac{\cos\frac{\vartheta}{2}}{\sin\frac{\vartheta}{2}} \cdot \frac{1}{2\sin\vartheta\cos\frac{\vartheta}{2}}$$

$$\frac{db}{d\vartheta} = K\frac{\sin\frac{\vartheta}{2}\left(-\sin\frac{\vartheta}{2}\right)\cdot\frac{1}{2} - \cos\frac{\vartheta}{2}\cos\frac{\vartheta}{2}\cdot\frac{1}{2}}{\sin^2\frac{\vartheta}{2}} = -\frac{K}{2}\frac{1}{\sin^2\frac{\vartheta}{2}}$$

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4\frac{\vartheta}{2}}$$

Semiempirična masna formula

Masa jedra $\quad m(A,Z)c^2 = N m_n c^2 + Z m_p c^2 + E_v(A,Z) < \cancel{0}$

Masa atoma $\quad M(A,Z)c^2 = N m_n c^2 + Z m_e c^2 + E_v(A,Z)$

The dudes in the past so si izmislili: model jedra kot "kapljevino" nukleonov, ki jih drži skupaj - jedrska sila

Vezavna en. je sestavljena z

prejmjiš izpeljano, el. potencial kroglice s homogenim ρ

$$E_v(A,Z) = -w_0 A + w_1 A^{\frac{2}{3}} + w_2 \frac{Z^2}{A^{\frac{1}{3}}}$$

↳ površinska napetost

$$+ w_3 \frac{(A-2Z)^2}{A}$$

↳ posledica KM

$$+ w_4 \frac{\delta_{ZN}}{A^{\frac{3}{4}}}$$

Vrednosti $\delta_{ZN} = \begin{cases} 1; & (Z \text{ lih}, N \text{ lih}) \\ 0; & (Z \text{ sod/lih}, N \text{ lih/sod}) \\ -1; & (Z \text{ sod/lih}, N \text{ sod, lih}) \end{cases}$

Za splošna jedra (srednje težka jedra):

$$\boxed{\begin{array}{ll} w_0 = 15'6 \text{ MeV} & w_4 = 33'5 \text{ MeV} \\ w_1 = 17'3 \text{ MeV} & \text{Vir: Strnad} \\ w_2 = 0'70 \text{ MeV} & \\ w_3 = 23'3 \text{ MeV} & \end{array}}$$

Zorko 5.2

$$M\left({}^{15}_{7}N\right) = 15'000108\,u$$

$$M\left({}^{15}_{8}O\right) = 15'003605\,u$$

$$M_H = 1{,}007825\,u$$

$$m_n = 1'008655\,u$$

$$u = 931{,}494\ \mathrm{MeV}/c^2$$

Kisik: $M\left({}^{15}_{8}O\right)c^2 = 7\,m_n c^2 + 8 M_H c^2 + E_v(15,8)$

Dušik: $M\left({}^{15}_{7}N\right)c^2 = 8\,m_n c^2 + 7 M_H c^2 + E_v(15,7)$

E_v: $-w_0 A \xRightarrow{A_O = A_N} E_{v_0}(O) = E_{v_0}(N)$

$$w_1 A^{\frac{2}{3}} \longrightarrow E_{v_1}(O) = E_{v_1}(N)$$

$$w_2 \frac{Z^2}{A^{\frac{1}{3}}} \longrightarrow E_{v_2}(O) \neq E_{v_2}(N)$$

$$w_3 \frac{(A-2Z)^2}{A} \longrightarrow E_{v3}(O) = E_{v3}(N)$$

$$(-1)^2 \qquad (1)^2$$

$$w_4 \frac{\delta_{ZN}}{A^{\frac{3}{4}}} \xrightarrow[\text{komb.}]{\text{liha-soda}} \delta_{ZN}(O) = \delta_{ZN}(N) = 0$$

$$\Rightarrow E_{v4}(O) = E_{v4}(N) = 0$$

$$\left(M\left({}^{15}_{8}O\right) - M\left({}^{15}_{7}N\right)\right)c^2 = M_H c^2 - m_n c^2 + \frac{w^2}{A^{\frac{1}{3}}}\left(\underbrace{Z_O^2 - Z_N^2}_{8^2 - 7^2 = 15}\right)$$

$A \to 15$

$$\Rightarrow w_2 = \frac{\left[c^2 M\left({}^{15}_{8}O\right) - c^2 M\left({}^{15}_{7}N\right) - M_H c^2 + m_n c^2\right] A^{\frac{1}{3}}}{Z_O^2 - Z_N^2} \doteq 0'58 \, MeV$$

Zorko 5.8 // stabilnost jeder na α-razpad

$A = 2Z$

Splošna formula ${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + \underset{\alpha}{{}^{4}_{2}He}$ // samo jedra

Stabilnostni kriterij

$$E_v(A,Z) \leq \underbrace{E_v(A-4, Z-2)}_{\text{razvijali bomo to}} + E_v(4,2)$$

Predpostavimo $A, Z >> 4$

$$E_v(A-4, Z-2) = \left.\frac{\partial E_v}{\partial A}\right|_{A,Z} \underset{(-4)}{dA} + \left.\frac{\partial E_v}{\partial Z}\right|_{A,Z} \underset{(-2)}{dZ} + E_v(A,Z)$$

// razvoj

// vstavimo v neenačbo

$$\emptyset \leq \left.\frac{\partial E_v}{\partial A}\right|_{A=2Z}(-4) + \left.\frac{\partial E_v}{\partial Z}\right|_{A=2Z}(-2) + E_v(4,2)$$