5.8

$A = 2Z$

Stabilnost na α razpad

$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + \alpha \; ({}^{4}_{2}He) $$

Razpad α se zgodi, ko velja $E_v(A, Z) \leq E_v(A-4, Z-2) + E_v(4,2)$

A in Z sta velika in ju razvijemo

$$ \Rightarrow 4\frac{\partial E_v}{\partial A} + 2\frac{\partial E}{\partial Z} - \overbrace{E_v(4,2)}^{\text{številka, ki jo poračunamo}} \leq \emptyset $$

SEMF: $E_v(A,Z) = -w_0 A + w_1 A^{\frac{2}{3}} + w_2 \frac{Z^2}{A^{\frac{1}{3}}} + \cancel{w_3 \frac{(A-2Z)^2}{A}} + \cancel{w_4 \frac{\delta_{ZN}}{A^{\frac{3}{4}}}}$

upoštevamo pogoj iz navodil

majhnu, na koncu dokažemo, da drži $\to \emptyset$

// upoštevamo koeficiente za splošna jedra

$$ \frac{\partial E_v}{\partial A} = -w_0 + \frac{2}{3} w_1 A^{-\frac{1}{3}} - \frac{1}{3} w_2 \frac{Z^2}{A^{\frac{4}{3}}} $$

$$ \left.\frac{\partial E_v}{\partial A}\right|_{A=2Z} = -w_0 + w_1 \frac{2^{\frac{3}{3}}}{3} Z^{-\frac{1}{3}} - \frac{1}{3} w_2 Z^{\frac{2}{3}} \frac{1}{2^{\frac{4}{3}}} $$

$$\frac{\partial E_v}{\partial Z} = 2 w_2 \frac{Z}{A^{\frac{1}{3}}}$$

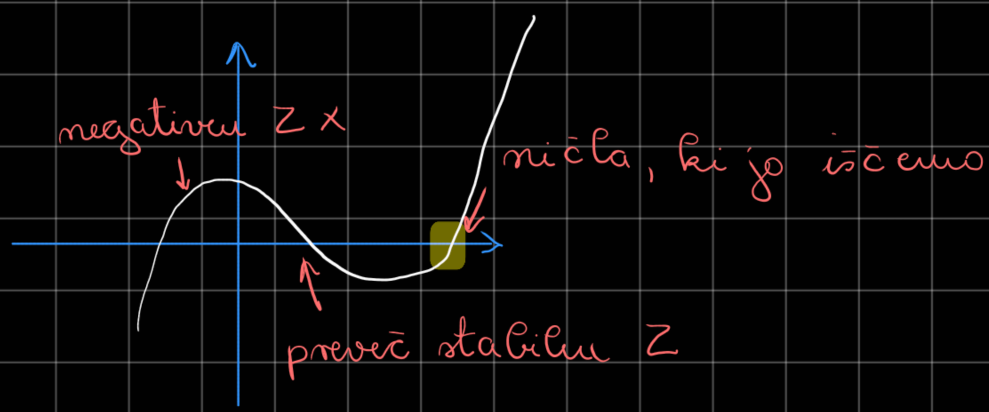
$$\left.\frac{\partial E_v}{\partial Z}\right|_{A=2Z} = 2^{\frac{2}{3}} w_2 Z^{\frac{2}{3}}$$

$$\underbrace{-4 w_0 + \frac{2^{\frac{8}{3}}}{3} Z^{-\frac{1}{3}} - \frac{2^{\frac{2}{3}}}{3} w_2 Z^{\frac{2}{3}}}_{\frac{\partial E_v}{\partial A}} + \overbrace{2^{\frac{5}{3}} w_2 Z^{\frac{2}{3}}}^{\frac{\partial E_v}{\partial Z}} - E_v(4,2) \leq 0$$

Neenačbo pomnožimo z $Z^{\frac{1}{3}}$

$$\underbrace{-4 w_0 Z^{\frac{1}{3}} + \frac{2^{\frac{8}{3}}}{3} w_1 - \frac{2^{\frac{2}{3}}}{3} w_2 Z + 2^{\frac{5}{3}} w_2 Z + E_v (4-2) Z^{\frac{1}{3}} \leq 0}_{I}$$

Kubična enačba, ki ima 3 ničle. V navodilih piše, da je ena ničla negativna



Naša ničla je med $40 \leq Z \leq 50$

Iterativno rešujemo

$$Z\left[2^{\frac{5}{3}} w_2 - \frac{2^{\frac{2}{3}}}{3} w_2\right] = Z^{\frac{1}{3}}\left[4 w_0 + E_v(4,2)\right] - \frac{2^{\frac{3}{3}}}{3} w_1$$

$$Z_{n+1} = \frac{Z_n^{\frac{1}{3}}\left[4 w_0 + E_v(4,2)\right] - \frac{2^{\frac{3}{3}}}{3} w_1}{2^{\frac{2}{3}}\left(2 - \frac{1}{3}\right)}$$

n ... indeks iteracije

Za začetek vstavimo $Z_0 = 50$

n	Z
0	50
1	46'9
2	45'5
3	44'8
⋮	
6	44'2

$\Rightarrow Z' = 44$

Po 6. iteraciji se spreminja samo še decimalke in $Z \in \mathbb{Z}$

Ne vemo, ali fja narašča ali pada in zato vstavimo

$Z = 43$ in $Z = 45$, v neenačbo

Vstavimo v I, $Z' + 1$ //nestabilna

$Z' - 1$ //stabilna

Jedra $Z > 44$ so nestabilna na α – razpad

Preverimo še $w_4 \frac{\delta_{ZN}}{A^{\frac{3}{4}}} = E_{v5}$ // pozna se v odvodu $\frac{\partial E_v}{\partial A}$

$$\Rightarrow 4 \left.\frac{\partial E_{v5}}{\partial A}\right|_{A=2Z} = \left. 4 w_4 \delta_{ZN} \left(-\frac{3}{4}\right) A^{-\frac{7}{4}} \right|_{A=2Z}$$

$$= -3 \cdot 2^{-\frac{7}{4}} w_4 \delta_{ZN} Z^{-\frac{7}{4}}$$

$$Z_{n+1} = \frac{Z_n^{\frac{1}{3}} \left[4 w_0 + E_v(4,2)\right] - \frac{2^{\frac{8}{3}}}{3} w_1}{2^{\frac{2}{3}} \left(2 - \frac{1}{3}\right)} + \alpha w_4 Z^{-\frac{17}{12}}$$

//dodamo člen

Popravki so $\delta_{ZN} = 1 \Rightarrow Z' = 44'4$

$\delta_{ZN} = -1 \Rightarrow Z' = 44'1$

Zorko 5.4

${}^{112}_{48}Cd$, ${}^{113}_{48}Cd$, ${}^{114}_{48}Cd$ $\quad u = 931'484 \frac{MeV}{c^2}$

$M({}^{112}_{48}Cd) = 111'902758 u$

$M({}^{113}_{48}Cd) = 112'904400 u$

$M({}^{114}_{48}Cd) = 113'903352 u$

Ne poznamo vrednosti w_i, $i = 0, 1, \dots 4$ $\approx$ Cd

$$E_v(A, Z) = -w_0 A + w_1 A^{\frac{2}{3}} + w_2 \frac{Z^2}{A^{\frac{1}{3}}} + w_3 \frac{(A-2Z)^2}{A} + w_4 \frac{\delta_{ZN}}{A^{\frac{4}{3}}}$$

$A \pm 1$

$E_v(A \pm 1, Z)$

upoštevamo:

- $(1+x)^\alpha = \sum_{m=0}^{\infty} \binom{\alpha}{m} x^m$ // do 1. reda razvijemo

$$\binom{\alpha}{m} = \frac{\alpha(\alpha-1)\dots(\alpha-m+1)}{m!} = \prod_{k=1}^{m} \frac{\alpha-m+1}{k!}$$

$$\binom{-\alpha}{m} = (-1)^m \binom{\alpha+m-1}{m}$$

Razvijemo člene

- $-w_0 A \xrightarrow{A \pm 1} -w_0 A \left(1 \pm \frac{1}{A}\right) \doteq -w_0 A (A \pm 1)$
- $w_1 A^{\frac{2}{3}} \longrightarrow w_1 (A \pm 1)^{\frac{2}{3}} = w_1 A^{\frac{2}{3}} \left(1 + \frac{1}{A}\right)^{\frac{2}{3}} \approx w_1 A^{\frac{2}{3}} \left(1 \pm \frac{2}{3} \frac{1}{A}\right)$
- $w_2 \frac{Z^2}{(A \pm 1)^{\frac{2}{3}}} \longrightarrow w_2 \frac{Z^2}{A^{\frac{2}{3}}} \frac{1}{\left(1 \pm \frac{1}{A}\right)^{\frac{2}{3}}} \approx w_2 \frac{Z^2 \left(1 \mp \frac{1}{A} \frac{1}{3}\right)}{A^{\frac{1}{3}}}$
- $w_3 \frac{(A-2Z)^2}{A}$

↳ imenovalec: $A^{-1} = (A \pm 1)^{-1} \approx A^{-1}(1 \mp A)$

↳ števec: $(A \pm 1 - 2Z)^2 = A^2 \left(1 \pm \frac{1}{A} - \frac{2Z}{A}\right)^2 =$

$$= A^2\left(1 \pm 2\left(\frac{1}{A}\right) - \frac{4Z}{A} + \left(\frac{1}{A}\right)^2 \mp \frac{4Z}{A^2} + \frac{4Z^2}{A^2}\right)$$

Združimo skupaj

2. red

$$\frac{A^2\left(1 \pm 2\left(\frac{1}{A}\right) - \frac{4Z}{A} + \cancel{\left(\frac{1}{A}\right)^2} \mp \frac{4Z}{A^2} + \frac{4Z^2}{A^2}\right)(1 \mp A)}{A} =$$

$$\frac{(A-2Z)^2}{A} \pm \boxed{\frac{2\left(\frac{1}{A}\right)A^2}{A} - \frac{4Z}{A^2}\,\frac{A^2}{A}} + \boxed{\frac{A^2}{A}\left(\mp\frac{1}{A}\right) + \frac{A^2}{A}\left(-\frac{4Z}{A}\right)\left(\mp\frac{1}{A}\right)}$$

$$\boxed{+\frac{A^2}{A}\left(\mp\frac{4Z}{A^2}\right)\left(\mp\frac{1}{A}\right) + \frac{A^2}{A}\left(\frac{4Z^2}{A^2}\right)\left(\mp\frac{1}{A}\right)} =$$

//množimo z $\frac{1}{A}$ samo tiste člene, ki nimajo že $\frac{1}{A}$

$$= \frac{(A-2Z)^2}{A} \pm 2 \mp \frac{4Z}{A} \mp 1 \pm \frac{4Z}{A} + \frac{4Z}{A^2} \mp \frac{4Z^2}{A^2}$$

Možno razviti w_3 kot

$$\frac{A^2 - 4AZ + Z^2}{A} = A - 4Z + \frac{Z^2}{A}$$ ki se sedaj razvije enostavnejše